Approximations for detection of periodic signals in image sequences

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ABSTRACT

This article describes our extended and generalized approach to detection of periodic signals in image sequences. These signals appear in a small number of pixels of an image sequence as periodic fluctuations in the temporal domain. Neither the shape of a signal, nor its fundamental frequency are assumed to be known, but the fundamental frequency is assumed to be localized in some narrow range. The frame sequences cover only a few periods of each signal under discussion. We consider three groups of these signals relative to our uniform sampling operator, that is defined by its frequency and integration (aperture) time. For each group the appropriate coherent basis is used: Fourier basis, periodized delta-functions, periodized gaussians or periodic wavelets. Not unusually, under the sampling operator the signals and basic functions loose periodicity, and the bases loose orthogonality. The problems that arise are treated by some version of matching pursuit. Our approach to signal accumulation from adjacent pixels by spectrum-specific version of principal components is generalized by using projection onto more general class of subspaces. Normally, the computationally expensive processing sketched above is performed for less than 1% of pixels only. The remaining 99% are rejected by simple and fast procedures. The algorithm was tested by processing simulated image sequences, as well as several real ones.

Keywords: image sequence, signal detection, power spectrum, coherent basis, principal component

1. INTRODUCTION

This article continues our work on signal detection in image sequences. The signals are in no sense synchronized with the sampling procedure. Specifically, the work deals with the problem of detecting tiny blobs, of few-pixel or even sub-pixel size, that pulse with some constant period in the temporal domain, in image sequences corresponding to uniformly sampled analog video. This period (or equivalently the corresponding fundamental frequency ) is assumed to be within some known narrow range, but neither its precise value nor the shape of the signal is assumed to be known. The video is assumed to be non-interlaced and uniformly sampled, this being the only restriction. In the present work, the method is extended to be applicable as well for detection of signals localized in the temporal domain better, than in the frequency domain. This involves the use of another "dictionary" for obtaining sparse representations of such signals. We construct such dictionaries by periodizing gaussian windows, but these dictionaries are huge and quite redundant, especially after sampling operator application. So we are forced to use some modification of coherent matching pursuit procedure, described in . Still the procedure remains computationally expensive, so it should be applied to the small amount of signals so that at least some preliminary selection is unavoidable. This preliminary selection procedure is also described. This step could exclude about 99% of pixels.

Usually, the information of a signal is contained in a few adjacent pixels even if the source is of sub-pixel size. This problem can arise due to atmospheric turbulence and (or) camera mis-focus, see for example . It arises also when the raster scan procedure duplicates scanning of each point in horizontal or vertical or both directions. In the first case, we could attempt some de-convolution algorithm to restore the signal. However, this is not the most efficient approach, because the detection, in principle, does not demand the restoration of a Point Spread Function, nor of the signal itself. We overcome this difficulty, as well as difficulties that arise from the need to detect weak signals, by accumulating the signal information from adjacent pixels before detection. For signals with sparse representation in frequency domain it was performed by some Spectrum-Specific version of Principal Components. It differs from the usual Principal Components in covariance matrix computation, in that we use "Spectrum-Specific Covariance", defined as an integral

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over a smaller subset in the frequency domain that contains the signal harmonic components. In the present paper we generalize these notions to basis-specific ones.

The structure of the article is as follows: in Ch. 2 we describe a model for signal sampling for a pixel level. In Ch. 3 we describe our coherent matching pursuit-like procedure to extract from a data periodic temporal signals contained in it. The dictionaries we use are those of 1) Fourier bases, and 2) Gabor-like bases. We describe these bases and their images under sampling operator as well. The bases depend on the unknown fundamental frequency, which is varied over the range $\omega_1 \leq \omega_0 \leq \omega_2$ to provide the best fit for the data.

In Ch. 4, we define, for an arbitrary subspace $V$ of signals, the $V$-specific (carried by $V$) covariance, correlation and Principal Components. The first such Principal Component should accumulate signal information from adjacent pixels. Ch. 5 describes the flow chart. Ch. 6 contains a discussion of the results.

2. SIGNAL MODEL AT PIXEL LEVEL

Let

$$f(t) = s(t) + \varepsilon(t) \quad t = t_0, t_1, \ldots, t_N \quad t_k = k \cdot \Delta t \quad \forall k$$

be the periodic real-valued signal $s_0(t)$ observed in the presence of the additive noise $\varepsilon(t)$. Unlike to 1, here we do not suppose, that the noise is white. Let us denote by $P_s$ the main period of $s_0(t)$ and by $Fr_s = 1 / P_s$ the corresponding "fundamental frequency". Now, let us suppose that $s(t)$ is the result of uniform sampling at the sampling frequency $Fr_{\text{det}}$ Hz, with the aperture (exposure) time $t_a$ msec. When

$$\Delta t = 1000 / Fr_{\text{det}} \, (\text{m sec}), \quad \text{and} \quad s(t) = \frac{1}{t_a} \int_{t}^{t+\varepsilon} s_0(\tau) d\tau \quad \forall t$$

Let us introduce the corresponding sampling function and the sampling operator as follows:

$$\text{samp}(t) = \begin{cases} 1 & t_k \leq t \leq t_k + t_a \quad \text{for some } k \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Samp}(s)(k) = \int_{k \cdot \Delta t}^{(k+1) \cdot \Delta t} \text{samp}(\tau) \cdot s(\tau) \cdot d\tau \quad k = 0, 1, \ldots, N$$

Suppose that a range of $N$ consecutive values is used for the signal detection. In 1, we supposed that

$$s(t) \approx s_0(t)$$

and approximated the signal on the range $[0, (N-1)\Delta t]$ by a trigonometric polynomial of small length $L$ by means of Linear List Squares applied to the sample

$$y_i = f(t_i), \quad i = 0, 1, \ldots, N - 1$$

The Fourier basis used was dependent on the single parameter, the unknown fundamental frequency $\omega_1 \leq \omega_0 \leq \omega_2$. It was found by some optimization on the finite greed. This approach appeared to be quite efficient when the signal has sparse representation with Fourier bases, that happens when the signal energy is well-localized in the frequency domain. If, to the contrary, it is well localized in the temporal domain, then by Heisenberg inequality (see 5)

$$\sigma_i^2(s) \cdot \sigma_m^2(s) \geq 1 / 4$$
that procedure does not lead to any sparse representation. In this case, the bases with better temporal domain energy localization should be used. For signals from such bases their support narrows, and could be comparable with the exposure time of our camera. So, (4) does not hold any longer, and we are forced to search for sparse representation

\[ s_0(t) = \sum_{k=1}^{L} a_k \cdot g_{\gamma_k}(t), \quad s(t) \approx \text{Samp}(s_0)(t) = \sum_{k=1}^{L} a_k \cdot \text{Samp}(g_{\gamma_k})(t), \quad f(t) \approx s(t) \]  \hspace{1cm} (6)

where \( \{g_{\gamma} \mid \gamma \in \Gamma\} \) is some basis of periodic functions.

3. COHERENT MATCHING PURSUIT-LIKE DETECTION STRATEGY

In this chapter we describe our coherent matching pursuit-like approach to periodic temporal signal detection at pixel level. The dictionaries we use are those of 1) Fourier bases, and 2) Gabor-like bases. These bases should not be multi-resolution ones, because the scale is approximately known, and these bases should be periodic ones. They should be time-translation invariant, but should not be frequency translation invariant by obvious reason. The Gabor atoms are employed, as usual, because of their optimal energy concentration. After description of the general procedure, we introduce our continuous dictionaries, then produce their images under our sampling operator, that could be highly redundant. So we choose grids to implement some kind of greedy strategy.

3.1 General procedure

Let \( D = \bigcup_{\lambda \in \Lambda} B^{\lambda} \) be some dictionary of bases. Its vectors should be of unit length but usually not orthogonal.

For each base \( B \) and for the dictionary \( D \) the correlation of a signal \( f \) with the base is defined as

\[ C(f, B) = \sup_{\gamma} \left| \left\langle f, g_{\gamma} \right\rangle \right|, \quad \text{if} \quad B = \{g_{\gamma} \mid \gamma \in \Gamma\}, \quad C(f, D) = \sup_{\lambda \in \Lambda} \left( C(f, B^{\lambda}) \right) \]  \hspace{1cm} (7)

The matching pursuit algorithm is the iterative procedure. At \( k \)-th step it finds the basis vector \( g_{\gamma_k} \), that best correlates with the \( k-1 \)-th residue:

\[ R^0 f = f, \quad R^{k-1} f = f - \sum_{j=1}^{k-1} \left\langle f, g_{\gamma_j} \right\rangle g_{\gamma_j} \quad (k > 0), \quad C(R^{k-1} f, g_{\gamma_k}) = \max \]  \hspace{1cm} (8)

Let \( C_B \) be the constant (dependent on \( B \) ) of correlation value with \( B \) on the attractor of unit length vectors, that do not correlate well with vectors from \( B \) (see 4). For example, if \( B \) is orthonormal basis for discrete signals of length \( N \), then \( C_B \) depends on \( N \) only 6:

\[ C_B = C_N = \frac{\sqrt{2 \log_2 N}}{\sqrt{N}} \]

The signal \( f \) is called noise relative to \( B \), if \( C(f, B) \leq C_B \). The iteration process is broken when the new residue is noise relative to \( B \). If it occurs after \( M \)-th iteration, then

\[ s_B = \sum_{j=1}^{M(B)} \left\langle f, g_{\gamma_j} \right\rangle g_{\gamma_j}, \quad e(B) = f - s_B \]  \hspace{1cm} (9)

is the approximation provided by the basis \( B \) and its error. Let \( B_f \) is so chosen, that
The corresponding approximation will be denoted $s_D$. The two-step minimization procedure is described below.

### 3.2 Continuous dictionary examples

We describe here two examples of continuous dictionaries. We do not make it frequency translation invariant, because we are concerned with the known range $\omega_1 \leq \omega \leq \omega_2$ only.

#### 3.2.1 Fourier dictionary

For each $\omega_1 \leq \omega \leq \omega_2$ set

$$B^\omega_{Fc} = \{ \phi_{k,a}(t) = \cos(2\pi \cdot k \cdot \omega \cdot t + a) \mid k \in [1,N/2]; a \in [0,\pi] \}$$

Each $B^\omega_{Fc}$ consists of $P^\omega = 1/\omega$ - periodic functions. The graph Fig.1a is an usual example. Set

$$D^{\text{Fourier},c}_{[\omega_1,\omega_2]} = \bigcup_{\omega_1 \leq \omega \leq \omega_2} B^\omega_{Fc}$$

#### 3.2.2 Periodized gaussian dictionary

For each $(a,b)$ set

$$g^{(a,b)}(t) = \begin{cases} \exp\left(-\frac{(t-a)^2}{2b^2}\right) & |t-a| < 3b \\ 0 & \text{otherwise} \end{cases}$$

For each $\omega_1 \leq \omega \leq \omega_2$ set

$$g^{(a,b)}_\omega(t) = \sum_{k=-\infty}^{+\infty} g^{(a,b)}(t + P^\omega \cdot k)$$

If $P^\omega = 1/\omega > 6b$, then only one summand for each $t$ could be non-zero. All these $g^{(a,b)}_\omega(t)$ are $P^\omega$ - periodic. The graph Fig.1b is an example of such a periodization. Set

$$B^\omega_{Gc} = \{ g^{(a,b)}_\omega \mid (a,b) \in \mathbb{R}^2, b < 1/6\omega \}$$

Finally, set

$$D^{\text{Gauss},c}_{[\omega_1,\omega_2]} = \bigcup_{\omega_1 \leq \omega \leq \omega_2} B^\omega_{Gc}$$

This dictionary is time translation invariant, but, unlike to the Gabor dictionary, is not frequency translation invariant and is not multiscale.

### 3.3 Discrete dictionaries

To approximate the results of sampling with a sampling operator (3), it is natural to use discrete dictionaries, obtained from the continuous ones by application of the same sampling operator:
Figure 1. a): Fourier basis element; b): periodized gaussian basis element; c): sampling function; d): the signal of a) sampled with the sampling function c); e): the signal of b) sampled with the sampling function c).
\[ B_{F}^{ao} = \text{Samp}(B_{Fc}) \quad D_{\text{Fourier}}^{ao} = \bigcup_{a_{1}, a_{2}} B_{F}^{ao} \quad (14a) \]

\[ B_{G}^{ao} = \text{Samp}(B_{Gc}) \quad D_{\text{Gauss}}^{ao} = \bigcup_{a_{1}, a_{2}} B_{G}^{ao} \quad (14b) \]

The examples of discrete basis functions obtained by sampling continuous functions from Fig.1a and Fig.1b with sampling function from Fig.1c are displayed in Fig.1d and Fig.1e. It should be mentioned, that the bases of (12a) are no longer orthogonal, while bases in (12b) are redundant.

### 3.4 Greedy strategy

Because of the inequality

\[ \|f_{1}, f_{2}\| \leq \|f_{1} - f_{2}\| \]

for each dictionary vectors \( f_{1}, f_{2} \) of unit length we obtain

\[ \|C(f_{1}, f_{2}) - C(f_{1}, f_{2})\| \leq \|f_{1} - f_{2}\| \]

So let us choose some small \( \delta \) and consider such a greed for \([\omega_{1}, \omega_{2}]\), that \( \|f_{1} - f_{2}\| < \delta \) if our dictionary vectors \( f_{1}, f_{2} \) correspond to the same parameters except \( \omega \), and their \( \omega \) are neighbors in the greed. The bases of this greed points form a sub-dictionary \( D^{\delta} \subset D \). At the first step of minimization the error (10) we choose a basis \( B_{ao} \in D^{\delta} \). Next, at the second step, we search in the sub-range around \( \omega \) whose radius equals the greed step, to improve approximation. For each \( B_{ao} \) approximation is performed as described in (8). Each best correlation is found by the similar two-step greedy strategy applied to a greed in basis parameter domain.

### 4. SIGNAL ACCUMULATION

The algorithm described here is the straightforward generalization of one described in 1. Let for some pixel \( p \) the algorithm, described at the previous chapter found the best approximation (9). The subspace generated by the basis vectors \( g_{i} \), we denote by \( V_{p} \), and the operator of the orthogonal projection onto this subspace we denote \( P_{p} \). We define covariance, correlation and principal components, corresponding to this subspace, as follows.

#### 4.1. Adaptive covariance

Let \( f, g \) be some signals and \( V \) be some subspace. We define covariance and correlation relative to the subspace as

\[ \text{cov}_{V}(f, g) = \langle P_{V}(f), P_{V}(g) \rangle \quad C_{V}(f, g) = C(P_{V}(f), P_{V}(g)) \]

We define our \( V \)-Specific Principal Components the same way as the usual ones, but using \( \text{cov}_{V} \) instead of \( \text{cov} \). Let \( f_{1}(t), ..., f_{n}(t) \) be some signals. Let us consider their \( V \)-specific covariance matrix:

\[ \text{cov}_{V}(f_{1}, ..., f_{n}) = \begin{pmatrix} \text{cov}_{V}(f_{1}, f_{1}) & \text{cov}_{V}(f_{1}, f_{2}) & \cdots & \text{cov}_{V}(f_{1}, f_{n}) \\ \vdots & \ddots & \vdots \\ \text{cov}_{V}(f_{n}, f_{1}) & \cdots & \text{cov}_{V}(f_{n}, f_{n}) \end{pmatrix} \]

The bi-linear function \( \text{cov}_{V}(-,-) \) is semi-positive, and the rank of the matrix above does not exceed \( m = \text{dim}(V) \).
Let \( \lambda_1, \lambda_2, \ldots, \lambda_m \) be the eigenvalues of \( \text{cov}_V(f_1, \ldots, f_n) \) listed in decreasing order and let \( v_1, v_2, \ldots, v_m \) be the eigenvectors of \( \text{cov}_V(f_1, \ldots, f_n) \) arranged in decreasing order of their associated eigenvalues. Let \( \Phi_V \) denote the matrix whose rows are these eigenvectors. The transform

\[
g' = \Phi_V \cdot f'
\]

is the \( V \)-specific Karhunen-Loeve transform, and the signals \( g_1, \ldots, g_k (k \leq m) \) are the first \( k \) \( V \)-Specific Principal Components of the initial set of signals \( f_1(t), \ldots, f_n(t) \).

4.2 Pixel signal accumulation from its neighbor signals
For pixel \( p \) we say that a signal \( g \) is related to \( f_p \), if

\[
C_V(f_p, g) \geq C_{B_p}
\]
where $C_{B_p}$ is the constant, described in the chapter 3.1 for the basis $B_p$ (the best basis for the pixel $p$). Let $p_1, p_2, \ldots, p_k$ be the set of all pixels, that are in some neighborhood of $p$ and are related to $p$. Let's construct the $V_{p_1}p_{2\ldots k}$ specific covariance matrix for $f_{p_1}, f_{p_2}, \ldots, f_{p_k}$ and zero each covariance if the corresponding correlation does not exceed $C_{B_p}$. The first $V_{p_1}$-specific principal component, $g_{p_1} = g_1$, will be our result of signal accumulation. It is, by its construction, some orthogonal linear combination of the neighbor signals, that carries maximal variance, produced by the same "coherent" structures, as $f_{p_1}$.

We exclude pixel $p$ from further processing if $C_{B_p}(f_p) < C_{N-p_{1\ldots k}}$, i.e. if even after making signal more stationary it does not correlate with itself better than white noise does.

5. FLOW CHART

The flow-chart of the algorithm is pictured in Fig.2. The approximation at the first stage could be performed as described in the chapter 3. The second time our approximation is simply the LS-improvement of that provided by our signal accumulation procedure, described in the chapter 4.

6. CONCLUSIONS

New algorithm for detection of periodic signals in image sequences was developed and tested. It generalizes our earlier approach 1. Our version of spectrum-specific principal components is generalized to coherent basis-specific one. The algorithm was tested by processing simulated signals as well as real ones and demonstrated a good efficiency.

REFERENCES

4. Ibid, chapter II, 2.3.
5. Ibid., chapter X, 10.5.